

# A critical review of multi-hole drilling path optimization

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## Abstract

Hole drilling is one of the major basic operations in part manufacturing. It follows without surprise then that the optimization of this process is of great importance when trying to minimize the total financial and environmental cost of part manufacturing. In multi-hole drilling, 70 % of the total process time is spent in tool movement and tool switching. Therefore, toolpath optimization in particular has attracted significant attention in cost minimization. This paper critically reviews research publications on drilling path optimization. In particular, this review focuses on three aspects; problem modeling, objective functions, and optimization algorithms.

We conclude that most papers being published on hole drilling are simply basic Traveling Salesman Problems (TSP) for which extremely powerful heuristics exist and for which source code is readily available. Therefore, it is remarkable that many researchers continue developing “novel” metaheuristics for hole drilling without properly situating those approaches in the larger TSP literature. Consequently, more challenging hole drilling applications that are modeled by the Precedence Constrained TSP or hole drilling with sequence dependent drilling times do not much research focus.

Sadly, these many low quality hole drilling research publications drown out the occasional high quality papers that describe specific problematic problem constraints or objective functions. It is our hope through this review paper that researchers' efforts can be refocused on these problem aspects in order to minimize production costs in the general sense.

*Keywords: Drilling process, Path optimization,*

## **1. Introduction**

Cost optimization of production processes remains one of the major focus points of machine builders world-wide. Machining in general and drilling in particular is one of the main production processes used to manufacture durable goods. Hole drilling is a process that uses a rotating drill bit to remove a circular cross-section of material from metallic or non-metallic materials. This process is a fundamental manufacturing process and thus is encountered in many industries and applications [1].

Given the fact that this process is so widely used, a great pressure exists to optimize the hole drilling process as much as possible. This can be achieved through better machine and tool design [2] and through process parameter optimization [3-6], but also through tool path optimization. Tool path optimization is the focus of this review paper. Non-cutting time can take up to 70% of the total time in the drilling process [7]. This includes repositioning times and tool switch times. Therefore, this is not an optimization problem that one can neglect without having significant impact on total production costs. Especially for the mass production systems, a small improvement on tool path can provide significant cost reductions for the companies. Therefore, there exist several studies in literature related with hole drilling.

Recently, Abidin et al. [8] composed an overview of papers published on hole-drilling path optimization between 1995 and 2017. They present an overview of publication trends, country origin and application areas. The discussion on problem modeling, objective functions, and optimization algorithms, however, does not provide many useful insights, neither for practitioners from industry nor academic researchers.

The purpose of this review paper is to give a clear overview of previous work on hole drilling in order to provide a clear approach on how to model and optimize hole drilling problems for the practitioner from industry or, for the academic scholars, a clear overview of remaining challenges in hole drilling path optimization.

Section 2 presents the hole drilling process in detail. Section 3 presents different approaches to model hole drilling processes. In section 4, the reviewed literature is discussed critically with regards to modeling approach, objective functions, and used algorithms. Finally, section 5 presents our conclusions and outlook for the future of hole drilling path optimization research.

## 2. Hole drilling process

The basic hole drilling process involves routing a single drill bit over a workpiece in such a way that all holes are visited in the fastest possible manner. Figure 1 shows a widely used basic example of a single tool hole drilling workpiece with 14 holes of the same diameter.

This is the most basic version of a multi-tool drilling applications are workpieces, where every hole has to be drilled by a single specific tool. We will refer to this problem as **single tool hole drilling (ST)**.

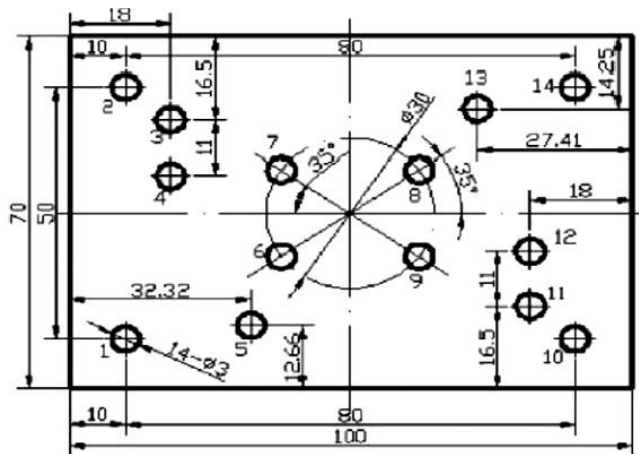


Figure 1 Generic small hole drilling work piece [42]

However, in reality, it is rare that all holes on a work piece require the same diameter or the same type of finishing. In that case, one speaks of multi-tool drilling and such a process attempt to minimize the sum of the tool switch costs and the tool travel costs. From an optimization point of view, however, this hole drilling problem with multiple tools reduces to the single tool hole drilling problem. Since, beforehand, we can define a simple cost matrix (or time matrix) that unambiguously defines the cost of moving from one hole to another hole. This cost equals the summation of the travel cost and the tool switch cost. At the lowest level, this is exactly

the same as the *single tool hole drilling* problem. However, we will refer to it as the basic **multi-tool hole drilling problem (MT)**

A more complex version is one where, for every hole, a specific sequence of tools is defined beforehand. In this case, tool switch costs need to be taken into account and the optimization algorithm needs to weigh travel costs against these tool switch costs. This is the case where, for examples, a work piece contains holes that first need to be predrilled all the way through before being finished by a tap or a reamer. We will refer to this type of drilling as **multi-tool hole drilling with precedence constraints(MT<sub>PC</sub>)**.

An even more complex version is the one presented by Kolahan and Liang [9]. In this hole drilling application with multiple tools, only the final tool for a given hole is known. However, for that hole, multiple smaller tools might be available to pre-drill the hole. Pre-drilling a hole with a smaller tool will reduce the time required to drill the hole with the larger tool as well as the wear on the larger tool. Figure two presents a small example where hole A might be drilled using tool sequences {3}, {1,3}, {2,3}, or {1,2,3}; hole C can only be drilled by sequence {1}; and hole B can be drilled by sequences {2}, or {1,2}.

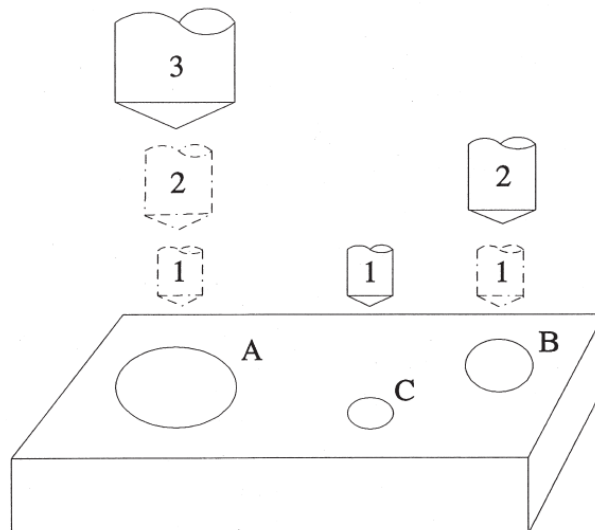


Figure 2 Example of multi-tool hole drilling with sequence dependent drilling costs and times [9]

This application involves four simultaneous optimization decisions: (a) tool-hole selection, (b) tool travel routing, (c) tool switch scheduling, and (d) selection of cutting speed for each tool-hole combination (operation) with the goal of minimizing total production cost. Total production cost consists of drilling cost, tool wear cost, tool travel cost, and tool switch cost.

In their work, Kolahan and Liang use numerical approaches to determine optimal cutting speeds for every tool-hole combination given a certain pre-drilled state of the hole (d). Therefore, (d) can be solved in a preprocessing phase to determine beforehand all cutting times and costs for every hole, drill, and pre-drill combination. Hence, in this multi-tool hole drilling problem with sequence dependent drilling times, the remaining optimization decisions are simultaneously (a), (b), and (c). We will refer to this problem as ***multi-tool hole drilling with sequence dependent drilling times ( $MT_{seq}$ )***.

The following section will discuss how these hole drilling applications are modeled as existing (well-studied) optimization problems.

### 3. Hole drilling models

In the following sub sections, we show how the ST hole drilling problem can be modeled as a Traveling Salesman Problem (TSP), how the MT hole drilling problem can be modeled as a Precedence Constrained Traveling Salesman Problem (PCTSP), and how the  $PC_{seq}$  hole drilling problem can be modeled as a Precedence Constrained Generalized Traveling Salesman Problem (PCGTSP).

#### 3.1 Single tool and basic multi tool hole drilling – TSP

The single tool hole drilling problem deals with drilling a set of holes on a work piece with a single drilling time using a single tool. The TSP is defined as: *given a set of cities with a priori known travel costs (or travel times, or distances) between any pair of cities, find the shortest tour that visits every city*. In the ST problem with a single tool, the cost matrix is evidently the distance, travel time, or travel cost between any two nodes.

In the basic multi tool problem, the cost between two nodes  $i$  and  $j$  consists of the travel cost between the two nodes plus the tool switch cost between the tool required to drill  $i$  and the tool required to drill hole  $j$ . It follows that the TSP model for the MT problem involves an asymmetric cost matrix. However, this does not matter much for the current state-of-the-art heuristic and exact TSP solvers.

Sherali et al. [10] discuss MIP formulations for the TSP and propose several new ones. The following MIP formulation is one (tight) way to formulate the TSP with  $n$  holes designated with index 1 to  $n$ .

Let  $x_{ij}$  equal 1 if the arc from hole  $i$  to hole  $j$  is selected, and 0 otherwise ( $\forall i, j = 1, \dots, n, i \neq j$ ). Let  $y_{ij}$  equal 1 if hole  $i$  precedes (not necessarily

immediately) hole  $j$ , and 0 otherwise ( $\forall i, j = 2, \dots, n, i \neq j$ ). Let  $c_{ij}$  equal the cost of moving from hole  $i$  to hole  $j$ . The TSP can then be formulated as follows:

$$\min \sum_{i=1}^n \sum_{j=1, j \neq i}^n c_{ij} x_{ij} \quad (1)$$

$$\sum_{v=1, v \neq i}^n x_{iv} = 1 \quad \forall i = 1, \dots, n \quad (2)$$

$$\sum_{i=1, i \neq v}^n x_{iv} = 1 \quad \forall v = 1, \dots, n \quad (3)$$

$$y_{ij} \geq x_{ij} \quad \forall i, j = 2, \dots, n, i \neq j \quad (4)$$

$$y_{ij} + y_{ji} = 1 \quad \forall i, j = 2, \dots, n, i \neq j \quad (5)$$

$$y_{ij} \geq x_{1i} \quad \forall i, j = 2, \dots, n, i \neq j \quad (6)$$

$$y_{ji} \geq x_{i1} \quad \forall i, j = 2, \dots, n, i \neq j \quad (7)$$

$$(y_{ij} + x_{ji}) + y_{jk} + y_{ki} \leq 2 \quad \forall i, j, k = 2, \dots, n, i \neq j \neq k \quad (8)$$

$$x_{1j} + x_{j1} \leq 1 \quad \forall i, j = 2, \dots, n, i \neq j \quad (9)$$

$$x_{ij} \in \{0,1\} \quad \forall i, j = 1, \dots, n, i \neq j \quad (10)$$

$$y_{ij} \geq 0 \quad \forall i, j = 2, \dots, n, i \neq j \quad (11)$$

Constraint set (2) ensures that all cities except for the end city are exited. Constraint set (3) ensures that all cities are except for the start city are entered. Constraint sets (4) to (9) are sub tour elimination constraints and simultaneously ensure that the  $y$ -variables correctly represent precedence relations between cities. Constraint set (9) forces the  $x_{ij}$  variables to be binary and, lastly, constraint set (10) in conjunction with the sub tour elimination constraints also ensures that the  $y_{ij}$  also take binary values.

State-of-the-art solvers are capable of optimally solving TSPs with thousands of cities [11]. However, this still requires substantial amounts of computation time, i.e. in the order of 1000 seconds for a 1000 city problem. Nevertheless, powerful heuristics exist and open source code is

available that find near-optimal solutions in very short computation times [12]. Helsgaun's improved Lin-Kernighan heuristic [12] routinely solves 1000 city TSP's to optimality in on average 11 seconds. Therefore, considering that problem sizes considered in hole drilling applications are limited to hundreds of holes (as opposed to thousands of cities in current academic challenges and benchmarks), we actually can consider the path optimization problem for ST and MT hole drilling as solved from a machine builder's perspective.

### 3.2 Multi-tool hole drilling with precedence constraints – PCTSP

Multi-tool hole drilling deals with drilling a set of holes on a work piece where a sequence of drilling operations for each hole is determined beforehand. For example, in figure 3a [13], hole 1 needs to be drilled by only tool 1, hole 2 first needs to be drilled by tool 1 and then by tool 2, and hole 3 needs to be drilled by tools 1, 2, and 3 in that order.

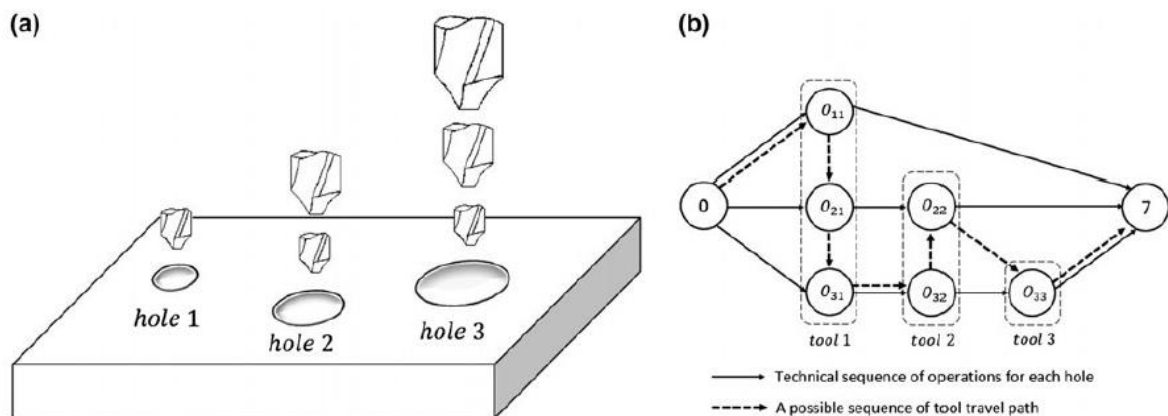


Figure 3 Multi-tool hole drilling with known tool-hole operations [13]

Figure 3b shows an operations precedence graph where nodes 0 and 7 represent the start and end of the hole drilling process. If one creates a *city* for every allowed hole-tool combination and one defines the cost of moving from a certain hole-tool combination  $(i,t)$  to another hole-tool combination  $(j,u)$  as the summation of the travel cost from hole  $i$  to hole  $j$  and the tool switch cost from tool  $t$  to tool  $u$ , the problem is actually identical to the Precedence Constrained Traveling Salesman Problem (PCTSP) [14] also known as the Sequential Ordering Problem (SOP) [15]. The PCTSP can be defined as follows: *given a set of cities, the costs of moving from one city to another city, and a set of precedence constraints between the cities, find the shortest path that visits every city without violating a precedence constraint.*

The TSP formulation of Sherali et al. [10] described above, can also be used to model the PCTSP since the  $y_{ij}$  variables denote whether a city  $i$  precedes another city  $j$ . Based on the precedence graph, certain  $y_{ij}$  variables can be fixed beforehand to 1 (and conversely, the corresponding  $y_{ji}$  variables can be fixed to 0).

Solving the PCTSP is significantly harder. This is showcased by the fact that still much research is being carried out in developing better exact approaches [14, 16], exploiting special cases [17], and investigating better heuristics [15, 18-22].

Skinderowicz [21, 22] developed the state-of-the-art Ant Colony Optimization – Simulated Annealing hybrid that is able to generate consistently very high quality solutions for problem instances with 200 to 700 nodes requiring 600 seconds of computation time. For many practical applications this is too high. On the other hand, many hole drilling applications do not deal with 200 holes and therefore such an approach could probably yield high quality solutions in shorter computation times. For the applications that deal with larger problem sizes, the current state of the art forces us to either accept the high computation times or to accept a lower solution quality.

Of course, the focus of academic researchers dealing with multi-tool hole drilling with precedence constraints should be to position their research within the larger PCTSP or SOP research field and to freely borrow and improve upon ideas present in the active research fields.

### **3.3 Multi-tool hole drilling with sequence dependent drilling times**

Based on our review of the hole drilling literature, the  $MT_{seq}$  as introduced by Kolahan and Liang [9] does not seem to be a very attractive problem. Only Dalavi, Pawar and Singh [23] and Dalavi [24] actually claim to deal with the  $MT_{seq}$  problem. Both their and Kolahan and Liang's solution approach seem to indicate that the problem structure does not easily translate into a standard Operations Research problem model. Kolahan & Liang and Dalavi, Pawar, and Singh represent a solution to the problem as a permutation of all possible tool-hole combinations and use a metaheuristic approach to generate neighbor solutions. Evaluating a single neighbor solution always requires a time complexity of  $O(n)$ , as opposed to evaluating a swap neighbor solution in a regular TSP which requires only  $O(1)$  time.



In the review paper of Dewil et al. [25] on tool path algorithms for laser cutting, it is suggested that the problem can be modeled as a Precedence Constrained Generalized Traveling Salesman Problem (PCGTSP), but a detailed modeling approach and computational experiments are still to be produced.

Therefore, the  $MT_{seq}$  problem is far from solved both from an academic point of view as from a practitioner's point of view with regards to modeling and efficient and easily implementable optimization approaches.

## 4 Discussion

### 4.1 Modeling approaches

Abidin, Ab Rashid and Mohamed [8] identify three models used in the literature on hole drilling: Traveling Salesman Problem (TSP), Traveling Cutting Tool Problem (TCP), and a so-called Precedence Sequence.

For the TSP model, Abidin, Ab Rashid and Mohamed [8] present a Mixed Integer Programming formulation which is actually incorrect since it does not contain sub tour elimination constraints. The TCP model is defined as a TSP problem where the tool head does not need to return to its starting position, tool changes are modeled as actual visits to a tool changing location and movements between holes might require additional moves to avoid collisions with the (static) work piece. It is remarkable that this is considered a separate problem since all of these issues can easily be preprocessed and taken into account in the regular TSP distance or cost matrix. The Precedence Sequence model is not explained in detail, but we assume it corresponds to the above defined PCTSP model.

Abidin, Ab Rashid and Mohamed [8] classify the modeling approaches of 41<sup>1</sup> reviewed papers as 92% TSP, 5% Precedence Sequence, and 3% TCP. Taking into account that the TCP actually is just a TSP problem, this means that 95% of hole drilling path optimization papers published between 1995

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<sup>1</sup> Abidin, Ab Rashid and Mohamed claim to review 61 papers on hole drilling path optimization, but going in detail over those publications, we were only able to identify 41 papers that deal with path optimization. More specifically, using the reference numbers of the paper of Abidin, Ab Rashid and Mohamed [8], [52] was omitted because of very poor quality and content, [60] described state of the art in optimization for maintenance system, [14, 15, 19, 21, 22, 23, 41, 42, 45, 62, 63, 64, 65, 66, 67] are general descriptions of metaheuristics not specifically applied to hole drilling, [51] was counted twice, and [5] deals with optimization of process parameters.

and 2017 attempt to solve the basic Traveling Salesman Problem. This seems a bit excessive.

Therefore, we prefer to use the above defined modeling approaches: TSP, PCTSP, and  $MT_{Seq}$ . We reviewed 53 papers on path optimization for hole drilling (including the 41 papers reviewed by Abidin, Ab Rashid and Mohamed) and augmented these with more recent or also relevant papers. These papers were published between 1998 and 2016. An overview of the number of publications by year is given in Figure 4.

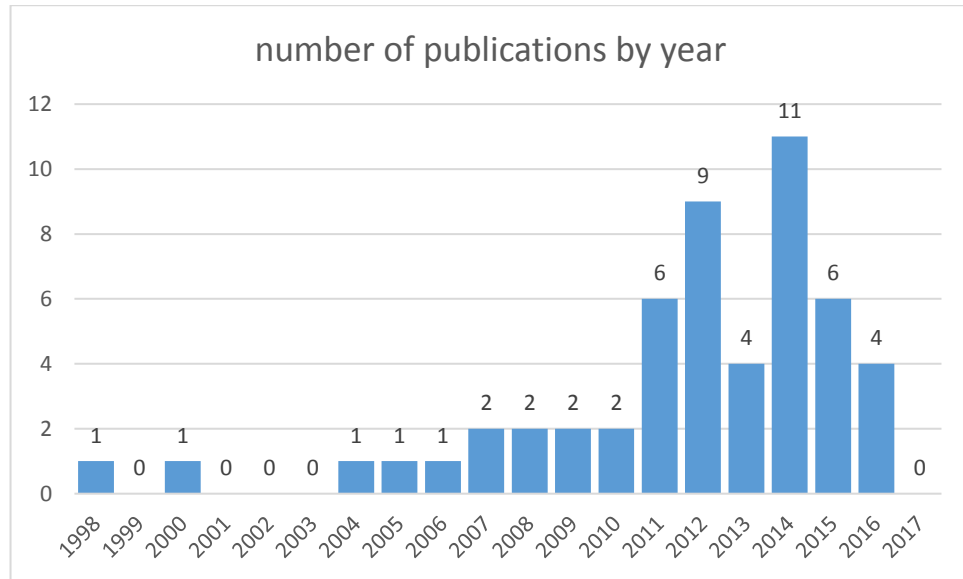


Figure 4 Overview of number of publications on hole drilling path optimization by year

As can be seen in Figure 5, 79% of papers (42) tackle the classical TSP, 13% (7) model the process as a PCTSP, and 8% (4) papers deal with the complex  $MT_{Seq}$  problem. Note that out of the 42 TSP papers, 38 deal with a single-tool hole drilling process and 4 with a basic multi-tool hole drilling process.

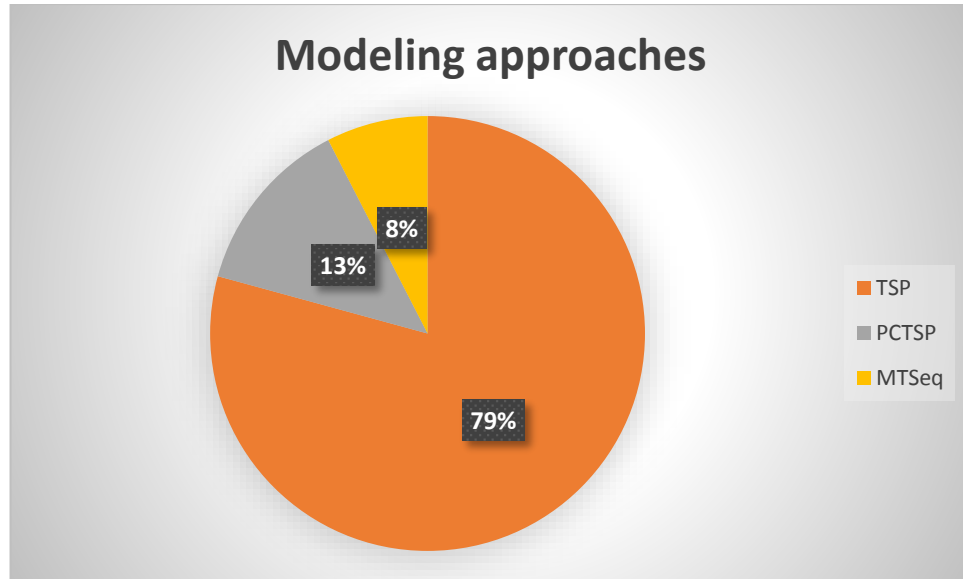


Figure 5 Overview of models used in hole drilling path optimization

Although, it is not 95% of papers, still a sizable 79% of papers develop custom TSP optimization algorithms while more powerful approaches can easily be found in previous work and in open source repositories.

Given the TSP-like nature of all problem types (ST, MT, MT<sub>pc</sub>, MT<sub>seq</sub>), solutions are represented as a permutation of tool-hole combinations (1A represents tool 1 – hole A), e.g. [1A, 2A, 6A, 3B, 2B, 4B, 5C].

Table 1 presents an overview of the reviewed papers. Columns 1 and 2 contain the reference number and publication year. Columns 3 and 4 contain the considered problem and the model used to approach the problem. Columns 5, 6, and 7 mark in which way distances or travel times are calculated, and column 8 contains the optimization algorithm(s) used.

Table 1 Overview of reviewed literature. GA: genetic algorithm, ACO: ant colony optimization, LKH: lin-kernighan heuristic, PSO: particle swarm optimization, TS: tabu search, BBO: biogeography based optimization, MOA: magnetic optimization algorithm, SFL: shuffled frog leaping algorithm

Reference	year	problem	model	distance matrix			algorithms
				Euclidean	rectilinear	Chebyshev	
26 Linn, Liu and Kowe	1999	ST	TSP	x	x	x	SA, nearest neighbor, sequential
27 El-Midany, Kohail and Tawfik	2007	ST	TSP	?	?	?	local search
28 Ancău	2008	ST	TSP	x			local search
29 Ancău	2009	ST	TSP	x			local search
30 Kentli and Alkaya	2009	ST	TSP	x	x	x	local search
31 Aciu and Ciocarlie	2014	MT	TSP	?	?	?	LKH
32 Abbas, Hamza and Aly	2011	ST	TSP	x			ACO
33 Saealal et al.	2012	ST	TSP	x	x	x	ACO
34 Medina-Rodríguez and Montiel-Ross	2012	ST	TSP	x	x	x	ACO
35 Montiel-Ross et al.	2012	ST	TSP	x	x	x	ACO
36 Eldos, Kanan and Aljumah	2013	ST	TSP	?	?	?	ACO
37 Abbas, Hamza and Aly	2014	ST	TSP	x			ACO hybrid
38 Guo et al.	2014	ST	TSP	x	x	x	ACO
39 Fathiyyah et al.	2014	ST	TSP	x			ACO

40	Naroei et al.	2014	MT	TSP	x			ACO
41	Abdullah et al.	2015	ST	TSP	x			ACO, GA
42	Zhu	2006	ST	TSP		x		PSO
43	Zhu and Zhang	2008	ST	TSP		x		PSO
44	Adam et al.	2010	ST	TSP		x		PSO
45	Othman et al.	2011	ST	TSP	x	x	x	PSO
46	Onwubolu and Clerc	2004	ST	TSP		x		PSO
47	Sigl and Mayer	2005	ST	TSP	x	x	x	GA
48	Katalinic	2011	ST	TSP	x	x	x	GA
49	Liu YC and Liu YB	2011	ST	TSP	x	x	x	GA
50	Chen and Sun	2012	ST	TSP	?	?	?	GA
51	Kumar and Pachauri	2012	ST	TSP	x			GA
52	Tsai, Liu and Wang	2012	ST	TSP	x	x	x	GA
53	Yang, Liu and Hung	2012	ST	TSP	x	x	x	GA
54	Qudeiri, Khadra, Al-Ahmari	2013	ST	TSP	x			GA
55	Nabeel, abid and Abdurazzaq	2014	ST	TSP	x			GA
56	Al-Janani and Liu	2014	ST	TSP		x	X	GA
57	Khalkar, Yadav and Singh	2015a	ST	TSP	x			GA
58	Tahir et al.	2010	ST	TSP	x			unspecified
59	Ismail et al.	2013	ST	TSP		X		Magnetic Optimization
60	Borkar, et al.	2014	MT	TSP	?	?	?	generic CAM heuristic
61	Alwis et al.	2015	ST	TSP	x			nearest neighbor, CAM heuristics
62	Yu and Shihtao	2012	ST	TSP	x	x	x	immune algorithm
63	Ismail et al.	2012	ST	TSP		x		Firefly Algorithm
64	Lim, Kanagarai and Ponnambalam	2014b	ST	TSP	x	x	x	cuckoo search
65	Kanagarai, Ponnambalam and Lim	2014	ST	TSP		x		cuckoo search/GA hybrid
66	Srivastava	2015	ST	TSP	x			intelligent water drops
67	Ghaiebi and Solimanpur	2007	MT <sub>PC</sub>	PCTSP		x		ACO
68	Hsieh et al.	2011	MT <sub>PC</sub>	PCTSP	x	x		PSO
69	Zhu and Chen	2011	MT <sub>PC</sub>	PCTSP			x	GA
70	Liu et al.	2013	MT <sub>PC</sub>	PCTSP		x		ACO
13	Tamjidy	2015	MT <sub>PC</sub>	PCTSP	x	x		BBO
71	Lim, Kanagarai and Ponnambalam	2014a	MT <sub>PC</sub>	PCTSP		x		cuckoo search/GA hybrid
73	Khalkar, Yadav and Singh	2015b	MT <sub>PC</sub>	PCTSP	x	x	x	GA
74	Chen and Guo	2016	MT <sub>PC</sub>	PCTSP	x	x	x	TS
9	Kolahan and Liang	2000	MT <sub>seq</sub>	MT <sub>seq</sub>	x			TS
23	Dalavi Pawar and Singh	2016	MT <sub>seq</sub>	MT <sub>seq</sub>		x		PSO, Shuffled Frog Leaping
24	Dalavi	2016	MT <sub>seq</sub>	MT <sub>seq</sub>		x		POS, SFL
75	Dalavi et al.	2016	MT <sub>seq</sub>	MT <sub>seq</sub>	x	x	x	N/A (review paper)

#### 4.1.1 MIP formulations

MIP formulations in PTCTSP and MT<sub>seq</sub> hole drilling literature are few and those MIP models that are formulated are in fact not subjected to computational tests. For example, Ghaiebi and Solimanpur [67] and Hsieh et al. [68] present MIP formulations with quadratic objective function for the PCTSP. Kolahan and Liang [9] present a MIP formulation which lacks sub tour elimination constraints for the MT<sub>seq</sub>. Abbas, Hamza and Aly [37] also present a MIP formulation for the TSP lacking sub tour elimination constraints.

In addition to the modeling issues of unnecessary development of TSP algorithms not advancing the state-of-the-art, seeing TCP as a separate problem as TSP, and the avoidance of looking for lower bounds using exact solvers are some of the indications that many publications on path optimization in the hole drilling literature are not well grounded in operations research techniques and models.

## 4.2 Optimization algorithms

As mentioned above, exact approaches are not being used in the reviewed papers. Researchers and practitioners use heuristics and metaheuristics to avoid the sometimes long calculation times of exact approaches. Figure 5 gives an overview of the algorithms used in the reviewed papers.

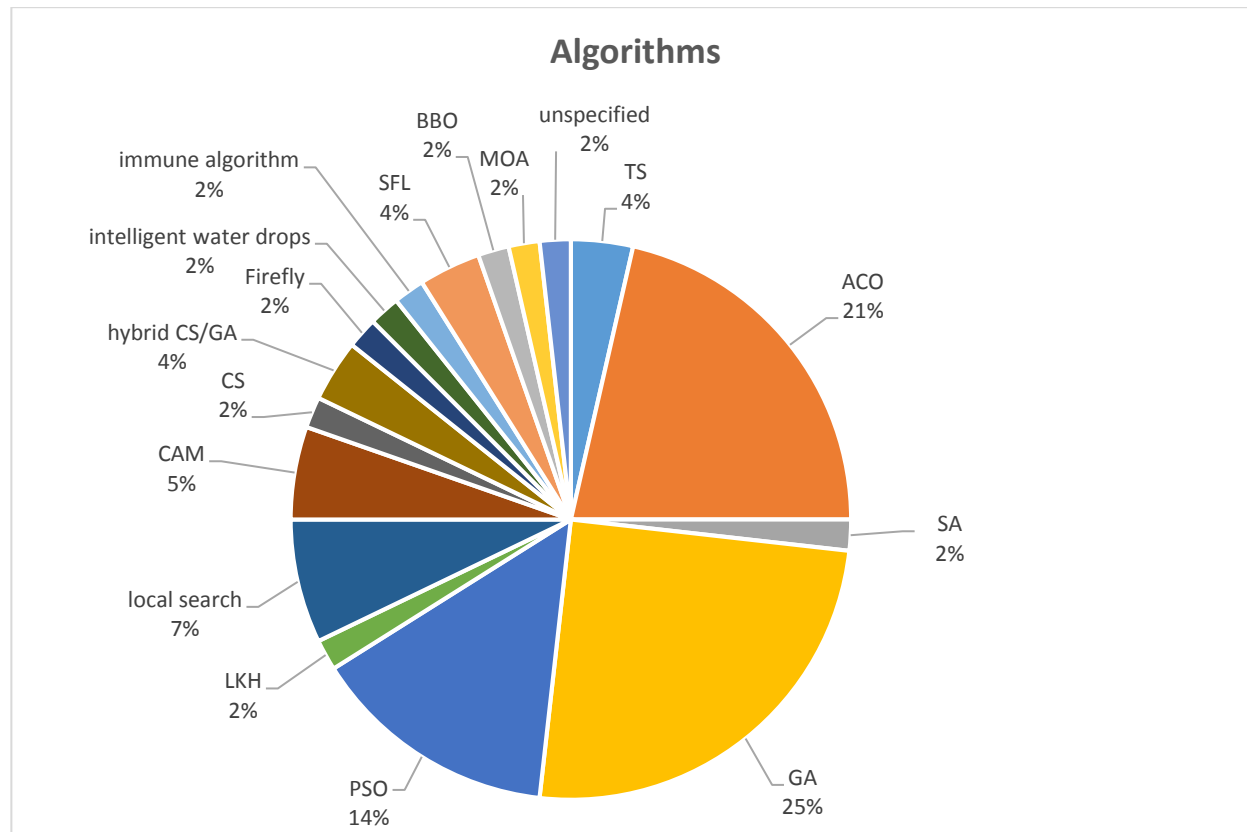


Figure 6 Overview of algorithms used in hole drilling path optimization

In the 53 reviewed papers, 56 algorithms were implemented. We can see that 75% of implemented algorithms use “classical” heuristics or metaheuristics: Tabu Search (TS), Ant Colony Optimization (ACO), Simulated Annealing (SA), Genetic Algorithms (GA), Particle Swarm Optimization (PSO), Lin-Kernighan Heuristic (LKH) and Local Search. 5% use or compare against basic CAM heuristics. 18% use so-called “novel” metaheuristics [72] and 1 paper did not give any details on the algorithms used.

Well over half of the optimization approaches (57%) use a population based approach (GA, PSO, CS, Firefly, Intelligent Water drops, Immune Algorithm, SFL, BBO, MOA). From an implementation perspective, this actually makes sense since these approaches can quickly be applied to problems that can be represented by a permutation of tool-hole combinations. In fact, for these approaches it does not matter much

whether the problem is a TSP, PCTSP, or  $MT_{seq}$  since the evaluation of new offspring, eggs, frogs, fireflies,... always takes  $O(n)$  time. There is no additional time complexity required to include precedence constraint checking or dealing with sequence dependent drill times. ACO, although not a population based approach, operates in a similar fashion: the process of generating a solution also ensures feasibility and is accompanied with the correct objective function value.

Tabu Search, Simulated Annealing, and Local Search, on the other hand, require a good understanding of the problem structure to define solution neighborhoods that can be searched efficiently for feasible solutions and evaluated efficiently. The advantage is that many more solutions are evaluated in the same time frame as population based algorithm. The disadvantage is that diversification requires additional explicit diversification mechanisms. Furthermore, other successful meta heuristics such as Variable Neighborhood Search and Large Neighborhood search have not been applied to the hole drilling problem before. These pose interesting avenues for further research since, at the very least, they require the development of different local move operators. Investigating which local move operators are successful is an interesting research in itself.

### 4.3 Objective functions

In the reviewed papers, paths are optimized for a single objective, being cost, distance, or time. Minimizing time or cost includes several or all of the following components: travel, drill, and tool switch times or costs, respectively.

As described above, for the ST, MT, and  $MT_{pc}$  problems, the total process cost can easily be captured in a two pre-computed cost matrices. The first containing the travel costs between any pair of holes and the second containing the tool switch costs between any pair of tools. In the ST, MT, and  $MT_{pc}$  problems, since all tool-hole combinations have been decided beforehand, no optimization of drill costs is possible and thus can be left out of the objective function.

The drill costs have to be included in the  $MT_{seq}$  problem and can also be captured in a simple 2 dimensional matrix where the cost in cell  $i,j$  corresponds to the cost to drill a hole with tool  $j$  when the hole is pre-drilled with tool  $i$ .

Travel costs are calculated as a function of travel distance or travel time. The reviewed papers use three different functions to model distance:

Euclidean, rectilinear and Chebyshev. Euclidean, rectilinear, and Chebyshev distances are calculated according to equations 12, 13, and 14 respectively.

$$d_{euclidean,ij} = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2} \quad (12)$$

$$d_{rectilinear,ij} = |x_1 - x_2| + |y_1 - y_2| \quad (13)$$

$$d_{chebyshev,ij} = \max(|x_1 - x_2|, |y_1 - y_2|) \quad (14)$$

To the best of the authors' knowledge, hole drilling machines use 2 separate motors for movements in the X and Y dimensions. Therefore, with regards to travel times or travel costs, a Euclidean distance matrix does not really make sense. It does, however, make the tool paths look *nicer* to a human observer.

A rectilinear distance matrix is applicable when the motors are activated in sequence, e.g. first execute the movement in X, and only then execute the movement in Y.

A Chebyshev distance matrix is applicable when both motors are activated simultaneously and, in that case, the longest distance is the one determining the actual travel time.

If one considers travel costs, there is an argument to be made that a combination of rectilinear and Chebyshev is actually most appropriate. The Chebyshev component captures travel time with its accompanying production cost per time unit and the rectilinear component captures the energy usage and wear and tear on the motors and their associated production costs per mm traveled. Such a combination is shown in equation 4. Where  $c_{time}$  is the production cost of the machine per time unit,  $v_x$  and  $v_y$  are the motor movement speeds in the x and y dimension respectively, and  $c_{motor}$  is the cost per mm movement of the x and y motors.

$$travel\ cost_{ij} = c_{time} \max\left(\frac{|x_1 - x_2|}{v_x}, \frac{|y_1 - y_2|}{v_y}\right) + c_{motor} d_{rectilinear,ij} \quad (15)$$

Such a travel cost component, however, is not used in the reviewed papers. Several papers repeat experiments with other distance metrics. In total, 34, 32, and 19 papers use Euclidean, rectilinear, and Chebyshev distances, respectively. Five papers do not specify which distance matrix they use.

Equation 15 assumes a constant speed in the x and y dimensions. Acceleration and deceleration, however, are a significant factor in the machine tool head movements given the powerful motors and the short distances involved. None of the reviewed papers considered the nonlinearities of acceleration and deceleration in setting up their distance matrix. From an academic point of view, this is understandable since all of the algorithms proposed in the reviewed papers function, regardless of the exact distance measure and travel time approximation used. From a practitioner's point of view, however, it does matter greatly. Since, the end goal is to minimize production costs and therefore, the algorithm should actually be minimizing the actual costs and not a simplification of these costs.

## 5 Discussion and future outlook

Based on the number of publications over the years, it would seem that hole drilling path optimization is a thriving research field. However, if one looks more closely, we see that 79% of papers (42 papers) deal with the basic TSP for which powerful heuristics are readily available. It would be better if research dealing with the basic TSP would be positioned within the TSP field.

Subtracting the TSP papers from the reviewed papers, we are left with 11 papers on hole drilling path optimization published between 1994 and 2017. Out of these 11, another 7 can be classified as the PCTSP or SOP which is a notoriously difficult problem. However, the reviewed papers on  $MT_{pc}$  validate their algorithms on very small problem instances. Liu et al [70] use a 42 hole problem, Khalkar, Yadav and Sing [73] use a 32 hole problem. Ghaiebi and Solimanpur [67] use a 10 and a 12 hole problem, Hsieh et al. [68] use a 12 hole problem, Chen and Guo [74] use a 6 and a 35 hole problem, and Tamjidy [13] considers the same 42 hole problem as Liu et al. and the same 10 hole problems as Ghaiebi and Solimanpur. It is remarkable that so much effort is spent on these small problems while SOP benchmark studies are being performed on instances with up to 700 cities [76]. Simply borrowing algorithms from these studies would mean a huge jump in cost savings for machine builders being confronted with  $MT_{pc}$  problems and would free up time from researchers to tackle unsolved problems.

Out of all reviewed papers, 4 papers deal with an actual not well understood problem, the hole drilling path optimization problem with sequence dependent drilling times or  $MT_{seq}$  in short. Current approaches



represent a solution as a single array containing all tool-hole combinations. Evaluation of a solution happens by iterating over a solution and possibly skipping a node if the hole has already been drilled to a larger size. It follows that such an evaluation is very practical for population based approaches or for constructive algorithms such as ACO. For local search based algorithms, such as the Tabu Search method using a swap local operator of Kolahan and Liang [9], the advantage of quick evaluations is not present and is not particularly well suited for this problem. Further research could focus on new solution representation techniques which could allow for quicker neighbor solution evaluation techniques in local search based metaheuristics. New specific local move operators could be developed to exploit the specific problem structure. And, although Kolahan and Liang described in detail what parameters were used to generate their instances, it would be useful to generate and make publicly available a set of many and large benchmark instances. Currently, no attempts have been undertaken to find exact solutions to  $MT_{seq}$  instances. Therefore, it would be very interesting to investigate different (linear) problem formulations and attempt to solve these using exact solvers.

## 6 Conclusions

Many publications on hole drilling path optimization have appeared over the years. This paper critically reviewed these publications and finds that 79% deal with the basic TSP problem and do not properly recognize the developments which have occurred over the years in the TSP path optimization field. These papers develop basic custom algorithms and frequently perform computational tests on very small problem instances. Such computational tests are of no or very low value for understanding the workings and limits of their proposed optimization approach.

More challenging optimization problems lie in 1) the PCTSP or SOP domains which can be used to model multi-tool hole drilling applications with precedence constraints, and 2) the  $MT_{seq}$  application which does not yet have a very convincing modeling approach or optimization algorithm.

Future research on hole drilling should focus on grounding the optimization models for  $MT_{pc}$  problems in the PCTSP or SOP literature, testing new algorithms on SOP benchmarks and large hole drilling instances.  $MT_{seq}$  problems are very challenging and developing a proper modeling approach and optimization strategy should be the main focus of researchers working on hole drilling path optimization. Such developments

would be of immediate interest to the industrial practitioner developing multi-hole drilling machines with sequence dependent drilling times.

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Figure 7 Generic small hole drilling work piece [42]

Figure 8 Example of multi-tool hole drilling with sequence dependent drilling costs and times [9]

Figure 9 Multi-tool hole drilling with known tool-hole operations [13]

Figure 10 Overview of number of publications on hole drilling path optimization by year

Figure 11 Overview of models used in hole drilling path optimization

Figure 12 Overview of algorithms used in hole drilling path optimization

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